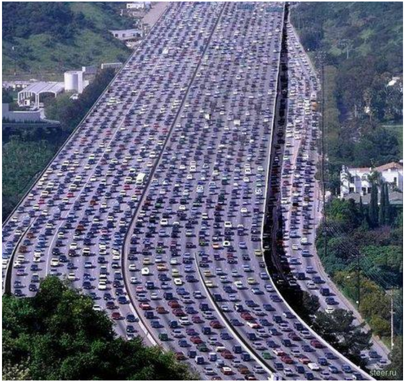
Resilient Control of Network Systems

Giacomo Como

Kick-off meeting of the DISMA Excellence Project
Torino, February 22, 2018

Fragility vs resilience in transportation networks

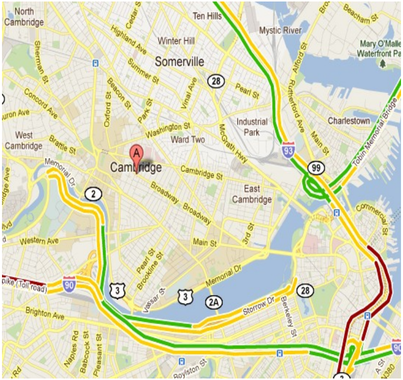


China Highway 110, August 2010, 10-days, 100 km-long queue

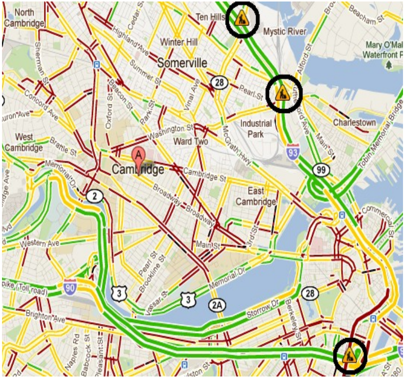
- often working close to infrastructure limits
- prone to disruptions: cascade effects

$\Longrightarrow \qquad$ network vulnerability $\gg \sum$ component vulnerabilities

Fragility vs resilience in transportation networks



typical Monday at 18:30

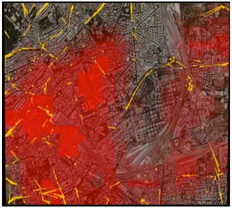


Monday, July 11, at 18:30

- often working close to infrastructure limits
- prone to disruptions: cascade effects

$\Longrightarrow$ network vulnerability $\gg \sum$ component vulnerabilities

Intelligent transportation networks




fast-increasing sensing and actuating capabilities



complex interactions between physics, cyber layer, and human behaviors

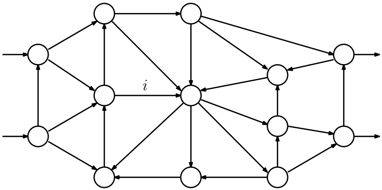
- scalable control with provable performance: efficiency + resilience

Resilience?

ability of the systems "to plan and prepare for, absorb, respond to, and recover from disasters and adapt to new conditions" [US-NAS]

- network system dynamics model
- measure of performance, minimal acceptable level
- (feedback) control policy
- set of perturbations
- "smallest" perturbation s.t. performance requirement not met

Dynamical flow networks



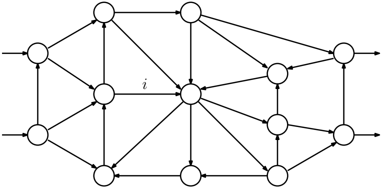
- $\mathcal{E}$ finite set of cells $\longleftrightarrow$ links[1] of a graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$
- $x_i = x_i(t) =$ volume in cell i

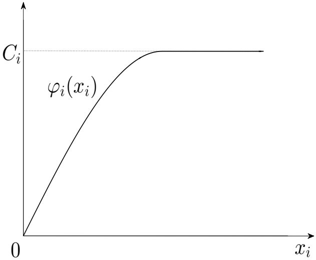
$$\dot{x}_i = \lambda_i + \sum_j R_{ji} z_j - z_i$$

- $\lambda_i = \lambda_i(t) \geq 0$ exogenous inflow in cell i
- $z_i = z_i(t) \geq 0$ total outflow from cell i
- $R_{ij} \geq 0$ fraction of outflow from i going to j
- $1 - \sum_j R_{ij} \geq 0$ fraction of z_i leaving network directly from i

[1] In other applications may be convenient to identify cells with nodes.

Dynamical flow networks



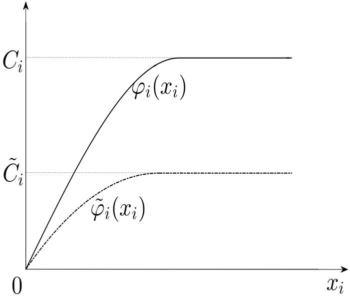


$$\dot{x}_i = \lambda_i + \sum_j R_{ji}(x) z_j - z_i$$

$$z_i = u_i(x) \varphi_i(x_i)$$

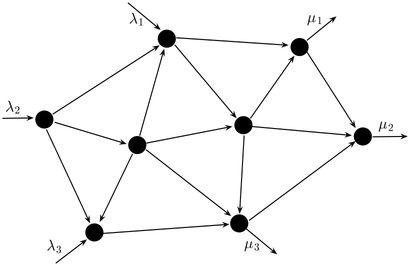
- $\varphi_i(x_i)$ max outflow, C_i link flow capacity
- $u_i(x) \in [0, 1]$ flow control
- $R_{ij}(x)$ dynamic routing

Measuring resilience



- perturbation of magnitude $\delta := \sum_i |\tilde{C}_i - C_i| + \sum_i |\tilde{\lambda}_i - \lambda_i|$

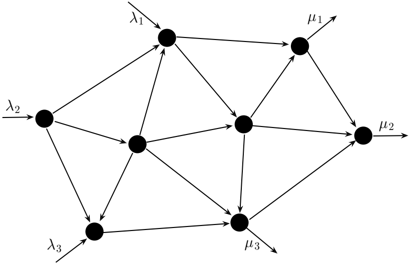
Measuring resilience



- perturbation of magnitude $\delta := \sum_i |\tilde{C}_i - C_i| + \sum_i |\tilde{\lambda}_i - \lambda_i|$
- perturbed system dynamics

$$\dot{x}_i = \tilde{\lambda}_i + \sum_j \tilde{z}_j R_{ji}(x) - \tilde{z}_i \qquad \tilde{z}_i = u(x_i)\tilde{\varphi}_i(x_i)$$

Measuring resilience



margin of resilience $\quad \nu := \inf \left\{ \delta : \sum_i x_i(t) \text{ unbounded} \right\}$

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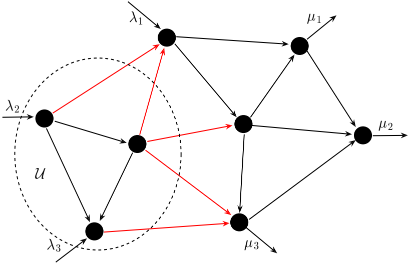
- $\delta :=$ magnitude of perturbation

Measuring resilience

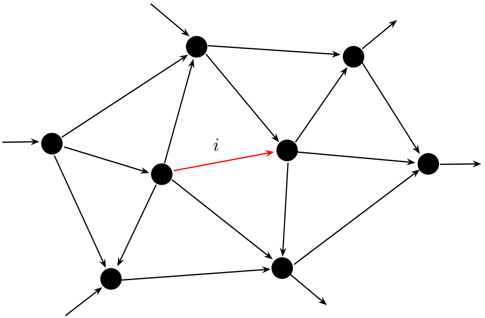


margin of resilience $\quad \nu := \inf\left\{\delta : \sum_i x_i(t) \text{ unbounded}\right\}$

$$\text{margin of resilience} \leq \text{min cut res. capacity} = \min_{\mathcal{U}}\{C_{\mathcal{U}} - \lambda_{\mathcal{U}}\}$$

- Problem: max ν over routing $R(x)$ and flow control $u(x)$
- What control 'architecture' is needed? Information flow?

Resilience with fixed routing



- $u_i \equiv 1$, $R_{ij}(x) \equiv R_{ij}$ constant
- start from equilibrium x^*, flow $z_i^* = \varphi_i(x_i^*)$

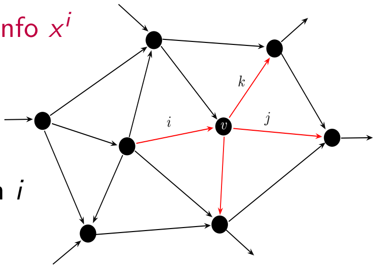
$$\nu = \underbrace{\min_i \{C_i - z_i^*\}}_{\text{min link residual capacity}}$$

Resilience with decentralized routing

(a) $u_i \equiv 1$, $R_{ij}(x^i)$ depends on local info x^i

$$R_{ij}(x^i) \geq 0 \qquad \sum_{j \in \mathcal{E}_i^+} R_{ij}(x^i) \equiv 1$$

$\uparrow$ set of links immediately downstream i



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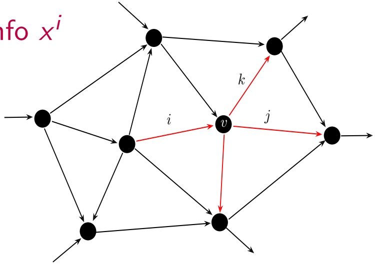

- Theorem [G.C.,K.Savla,D.Acemoglu,M.Dahleh,E.Frazzoli,TAC'13]

(a) equilibrium z^* $\implies$ margin of resilience $\leq \underbrace{\min_{v \text{ used}} \sum_{j \in \mathcal{E}_v^+} (C_j - z_j^*)}_{\text{min node res. capacity}}$

Resilience with locally responsive routing

(a) $u_i \equiv 1$, $R_{ij}(x^i)$ depends on local info

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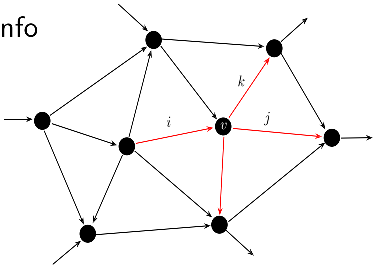

(b) $\dfrac{\partial}{\partial x_k} R_{ij}(x^i) \geq 0 \qquad \forall k \neq j$

$$x_j \to \infty \Rightarrow \ R_{ij} \to 0$$

- Example: i-logit

$$R_{ij}(x^i) = \frac{e^{-\beta(x_j+\alpha_j)}}{\sum_{k \in \mathcal{E}_i^+} e^{-\beta(x_k+\alpha_k)}} \qquad \beta > 0$$

$$1/\beta = \text{noise} \qquad \alpha_j = \text{a priori cost}$$

Resilience with locally responsive routing

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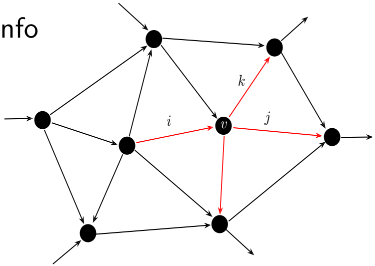

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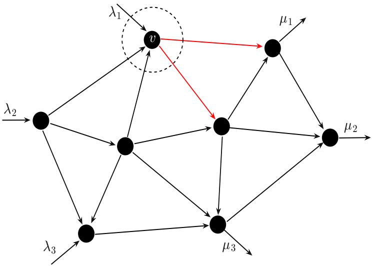
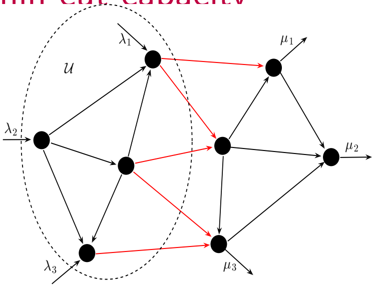
In acyclic networks

$(a) + (b) \quad \Longrightarrow \quad$ margin of resilience $\quad = \quad$ min node res. capacity

Min node residual capacity vs min-cut capacity





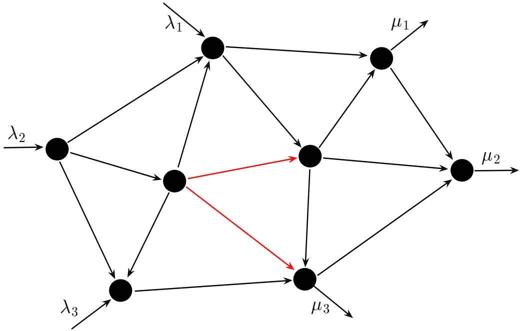
min node res. capacity $\leq$ min cut res. capacity

$\nearrow$ depends on equilibrium

$\nwarrow$ depends on inflows only

- the gap can be arbitrarily large (even for optimal equilibrium)
- min node res. capacity $\geq \alpha \iff$ linear inequalities (quasi-concave)

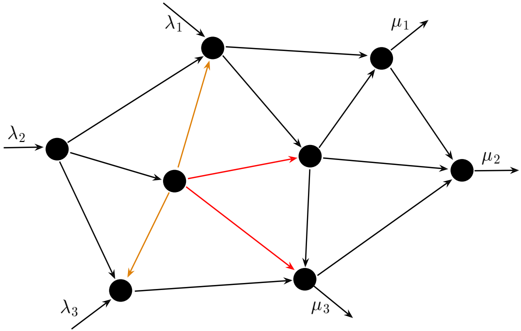
Resilience with locally responsive routing



margin of resilience = min node res. capacity

- perturbations and information propagate downstream only
- subadditive effects of perturbations

Resilience with locally responsive routing



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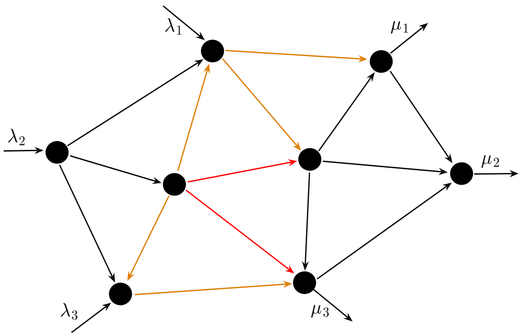
Resilience with locally responsive routing



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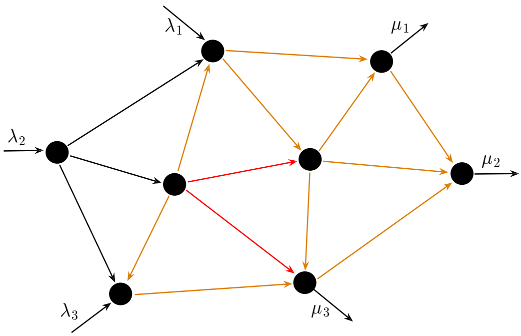

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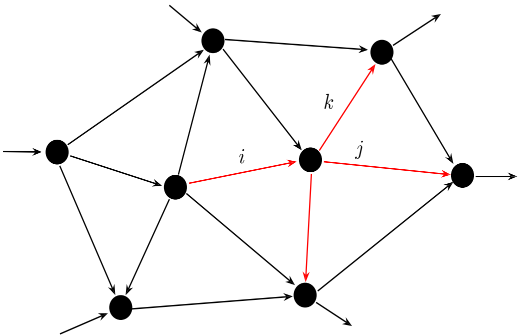
Is decentralized architecture preventing optimal resilience?



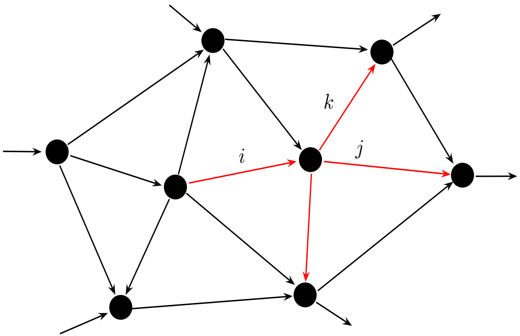
$$\dot{x}_i = \lambda_i + \sum_j R_{ji} z_j - z_i$$

$$R_{ij} = R_{ij}(x^i) \qquad z_i = \varphi_i(x_i)$$

$\nwarrow$

local information

Decentralized routing with flow control



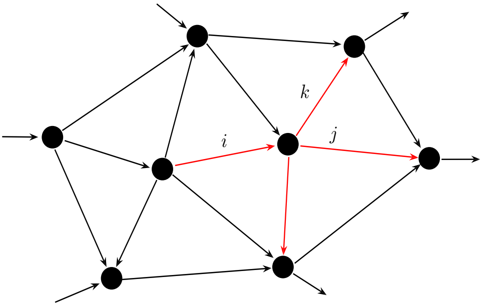
$$\dot{x}_i = \lambda_i + \sum_j R_{ji} z_j - z_i$$

$$R_{ij} = R_{ij}(x^i) \qquad z_i = u_i(x^i)\varphi_i(x_i)$$

$\nwarrow \qquad \nearrow$

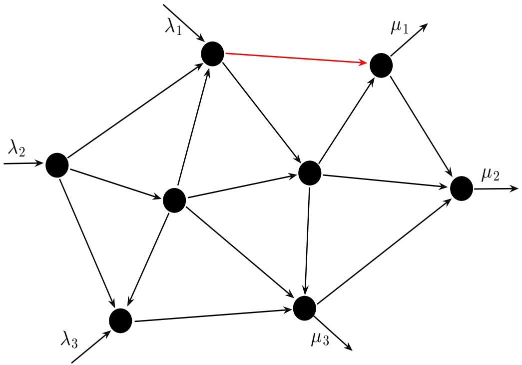
local information

Decentralized monotone routing with flow control



- idea: slow down flow when congestion downstream > upstream
- keep closed-loop system monotone + boundary conditions
- example: $$u_i(x^i)R_{ij}(x^i) = \frac{e^{-\beta(x_j+\alpha_{ij})}}{e^{-\beta(x_i+\alpha_{ii})} + \sum_{k\in\mathcal{E}_i^+} e^{-\beta(x_k+\alpha_{ik})}}$$

Decentralized monotone routing with flow control



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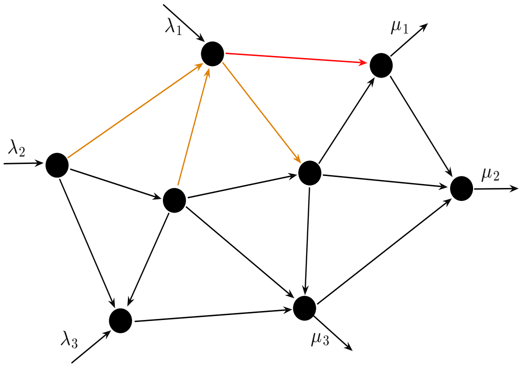
Decentralized monotone routing with flow control



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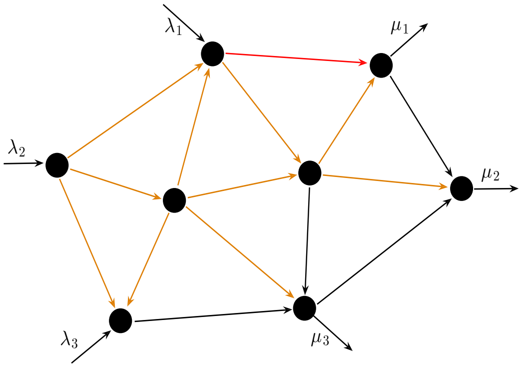

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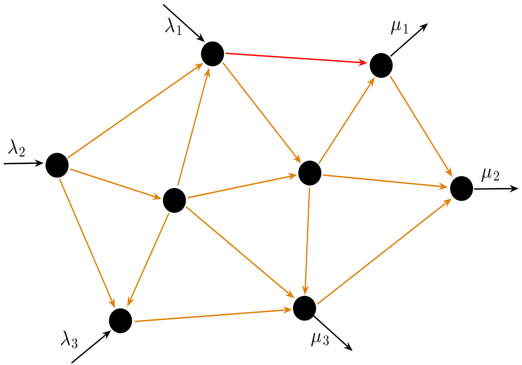
Decentralized monotone routing with flow control



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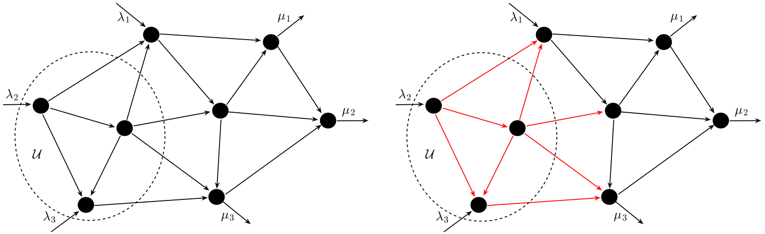
Decentralized monotone routing with flow control



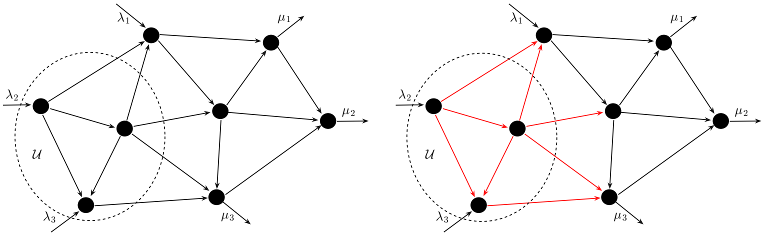
Theorem [G.C.,E.Lovisari,K.Savla,TCONES'15] :

- $\min_{\mathcal{U}}\{C_{\mathcal{U}} - \lambda_{\mathcal{U}}\} > 0 \Longrightarrow$ $\exists$ globally asymptotically stable equilibrium

margin of resilience $=$ min cut res. capacity

- $\min_{\mathcal{U}}\{C_{\mathcal{U}} - \lambda_{\mathcal{U}}\} < 0 \Longrightarrow$ minimal throughput loss, graceful degradation

Decentralized monotone routing with flow control

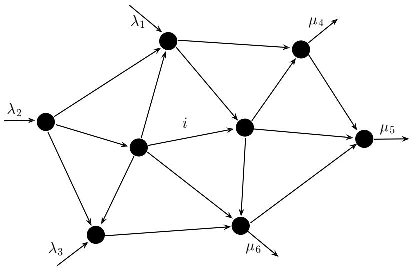


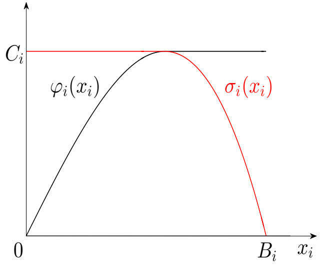
- decentralized routing + flow control achieve optimal throughput in a resilient way implicitly propagating information
- proof exploits monotonicity and l_1-contraction of closed-loop dynamics
- for other performance measures (e.g., total travel time or delay) communication / distributed optimization layer necessary

Many more open problems

in transportation networks:

▶ flow dynamics model too simple: should incorporate supply constraints to account for upstream shock propagation





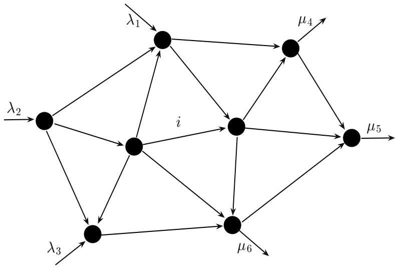
$$\dot{x}_i = \lambda_i + \sum_j R_{ji} z_j - z_i$$

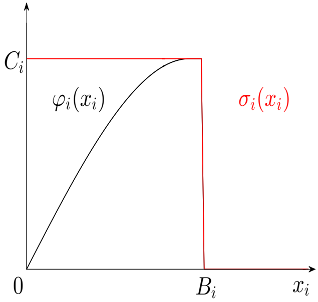
$$z_i \leq \varphi_i(x_i) \qquad \lambda_i + \sum_j R_{ji} z_j \leq \sigma_i(x_i)$$

Many more open problems

in transportation networks:

- flow dynamics model too simple: supply constraints
- different perturbation models, cascading failure mechanisms: coevolution of network and flow (hybrid system)





Many more open problems

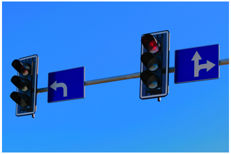
in transportation networks:

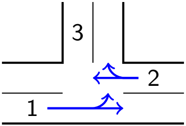
- flow dynamics model too simple: supply constraints
- different perturbation models, cascading failure mechanisms
- efficiency measures beyond throughput: equilibrium flows are not equally efficient, e.g., average delay, traffic volume, ...

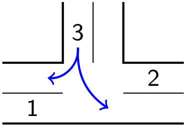
Many more open problems

in transportation networks:

- flow dynamics model too simple: supply constraints
- different perturbation models, cascading failure mechanisms
- efficiency measures beyond throughput
- decentralized scheduling for traffic signal control







Many more open problems

in transportation networks:

- flow dynamics model too simple: supply constraints
- different perturbation models, cascading failure mechanisms
- efficiency measures beyond throughput:
- scheduling for traffic signal control
- different control architectures and information flows: coupling physical system with "cyber" (computation/communication) layer

Many more open problems

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- flow dynamics model too simple: supply constraints
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- efficiency measures beyond throughput:
- scheduling for traffic signal control
- different control architectures and information flows
- estimation and learning from data: driver behaviors

Many more open problems

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- selfish behaviors, incentive mechanisms, selective information route-guidance

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Mathematical tools from graph theory, game theory, non-linear systems, convex optimization, robust and optimal control

Many more open problems

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- different perturbation models, cascading failure mechanisms
- efficiency measures beyond throughput:
- scheduling for traffic signal control
- different control architectures and information flows
- estimation and learning from data: driver behaviors
- efficient incentive mechanisms, selective information

and in other flow networks:

- production networks and supply chains
- distribution networks (energy, gas, water)